Threshold Resummation for Drell-Yan Process in Soft-Collinear Effective Theory

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Abstract

We consider Drell-Yan process in the threshold region $z \to 1$ where large logarithms appear due to soft-gluon radiations. We present a soft-collinear effective theory approach to re-sum these Sudakov-type logarithms following an earlier treatment of deep inelastic scattering, and the result is consistent with that obtained through a standard perturbative QCD factorization.

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I. INTRODUCTION

It has been known for many years [1] that next-to-leading order (NLO) calculations in perturbative quantum chromodynamics (pQCD) for cross sections of various hard scattering processes, lead to functions singular at the edge of the relevant phase space. For example, for Drell-Yan (DY) process we define $z = Q^2/s$ where Q^2 is the invariant mass of the produced lepton pair squared and s is the total partonic invariant mass squared. At nth order in pQCD we encounter terms like $\alpha_s^n[(1-z)^{-1}\ln^{2n-1}(1-z)]$. In the limit $z \to 1$ these singular functions—when Mellin transformed—give rise to large double logarithms in moment space $(\alpha_s \ln^2 N)^n$. When N is large, the product of these large logarithms with a small coupling constant α_s is not necessarily small and an all-order resummation in perturbation theory is needed to make predictions reliable. The purpose of a resummation is to control these large logarithms and to obtain a closed expression that sums part or all of these logarithms. For DY and deep-inelastic scattering (DIS) processes, an extensive theoretical work was performed in Refs.[2, 3], according to which, one has to establish first a factorized form of the cross section into a hard part ("Wilson coefficient"), jet part and soft part, and then make a clever use of an evolution in parton rapidity. We will have more to say about this in section IV.

Recently an effective field theory approach was implemented [4] to re-sum leading logs (LL) and next-to-leading logs (NLL) for DIS at large momentum transfer and near the threshold limit $x \to 1$ (where x is the longitudinal hadron momentum fraction carried by the quark). That analysis was performed within the soft-collinear effective theory (SCET) framework [5, 6, 7]. By utilizing this effective theory for the inclusive DIS (for large momentum transfer and near threshold) cross sections, one can re-derives the full QCD results (as a consistency check of the effective theory!) More importantly, the exponentiation of the LL and NLL in moment space appears quite naturally in this context following the solution of a simple RG equation [4].

In this work, we extend the application of SCET to the DY cross section in the threshold limit $z \to 1$ and show how the resummation is obtained. We work in the center-of-mass (c.o.m.) frame of the incoming quark and anti-quark, and identify three relevant scales: Q^2, $Q^2(1-z)^2$ and Λ_{QCD}^2. In the threshold limit we have $1 - z \ll 1$. Assuming $\Lambda_{\text{QCD}}^2/Q^2 \ll (1-z)^2$, we have a clear separation of scales. Given these three scales one has to perform a two stage matching and running: First, at Q^2 we match the full QCD (annihilation) current onto the SCET current, giving rise the so-called SCET$_I$ in which virtuality of order Q^2 is integrated out. Then we match, at the intermediate scale $Q^2(1-z)^2$, the SCET DY cross section onto a product of SCET parton distribution functions (PDF) [8], giving rise to SCET$_{II}$ in which virtuality or order $Q^2(1-z)^2$ is integrated out. Here we assume that the usoft interactions cancel as in the standard Drell-Yan factorization which is valid in the kinematic region of our interest [9]. The second matching yields the infrared-safe hard scattering coefficient function. Then, the PDF will run down in scale and be matched onto local twist-two operators.

This paper is organized as follows. In section II we give the basic notation, definitions and power counting of SCET. In section III we perform the two-stage matching and write down the anomalous dimensions. We compare our results for DY (as well as those of DIS) with the full QCD calculations. In section IV we derive the resummation (and exponentiation) of the large logs and compare our results with those given in [2, 3]. We conclude the paper in Section V.

II. SCET PRELIMINARIES

Soft-collinear effective field theory was invented to handle processes with multiple momentum scales [5, 6, 7]. This is the case in DY and DIS when final-state hadrons have an invariant mass much greater than Λ_{QCD} but much smaller than the highest momentum scale of the problem. In this section, we briefly describe some of the concepts and notations in SCET. We choose

$$n^\mu = \frac{1}{\sqrt{2}}(1,0,0,-1), \quad \bar{n}^\mu = \frac{1}{\sqrt{2}}(1,0,0,1), \quad n^2 = \bar{n}^2 = 0 \;. \tag{1}$$

For a generic four-vector l let $l^\pm \equiv \frac{1}{\sqrt{2}}(l^0 \pm l^3)$ and write $l \equiv (l^+, l^-, l_\perp)$. Then

$$l^+ = n \cdot l, \; l^- = \bar{n} \cdot l, \; l^2 = 2l^+l^- + l_\perp^2 = 2n \cdot l\,\bar{n} \cdot l + l_\perp^2 \;. \tag{2}$$

With this notation one has to trivially modify the SCET Feynman rules given in [5].

SCET describes the interaction of "collinear" quarks with "collinear" and/or "soft" gluons. If we have a fast-moving quark or gluon in the n-direction then we assign to its momentum components the following scaling: $p = (p^+, p^-, p_\perp) \sim Q(\lambda^2, 1, \lambda)$ where Q is some relevant hard scale and λ is a small parameter identified by considering the different momentum scales available. A quark with such an assignment is called n-collinear and is described by ξ_n field and n-collinear gluon has A_n^μ field. Similarly, for $\bar{n}$-collinear quark field we assign $p = (p^+, p^-, p_\perp) \sim Q(1, \lambda^2, \lambda)$ and is described by $\xi_{\bar{n}}$ field. For the DY case, and in the c.o.m. frame, we let the incoming quark be moving in the $+z$ direction (i.e p_1^+ is "large") and the incoming anti-quark moving in the $-z$ (i.e. p_2^- is "large"); then these fields are described by $\xi_{\bar{n}}$ and ξ_n respectively. At the scale Q, after integrating out virtuality of order Q^2, the electromagnetic current in $\mathrm{SCET_I}$ is given by

$$j^\mu = C(Q)\, \bar{\xi}_n W_n \gamma^\mu W_{\bar{n}}^\dagger \xi_{\bar{n}} \;. \tag{3}$$

The coefficient $C(Q)$ has been calculated in [4] and will be presented in the next section. The W_n and $W_{\bar{n}}^\dagger$ are the familiar path-ordered Wilson lines and are required to insure collinear gauge invariance of the current operator [6, 10]. This Wilson line is identical to the "eikonal" line introduced in [8]

$$W_n(x) = \mathrm{P} \exp\left[ig \int_{-\infty}^{x} \mathrm{d}s\, \bar{n} \cdot A_n(s\bar{n}) \right] \;, \tag{4}$$

where the covariant derivative is: $D_\mu = \partial_\mu - igT^a A_\mu^a$. A "soft" gluon has the momentum scale: $k = Q(\lambda^2, \lambda^2, \lambda^2)$. A SCET PDF for a $\bar{n}$-collinear field is defined as the matrix element of the operator

$$O_q(r^+) = \frac{1}{4\pi} \int_{-\infty}^{\infty} \mathrm{d}z\, e^{-izr^+} \left[\bar{\xi}_{\bar{n}} W_{\bar{n}}\right](nz)\, \slashed{n} \left[W_{\bar{n}}^\dagger \xi_{\bar{n}}\right](0) \;, \tag{5}$$

between $\bar{n}$-collinear quark fields. It is clear that for DY process one needs to consider the n-collinear version of Eq. (5), i.e., interchanging $\bar{n} \leftrightarrow n$ and taking the matrix element between n-collinear fields.

In this work we use the Feynman gauge and the $\overline{\mathrm{MS}}$ scheme in $d = 4 - 2\epsilon$. We will also use on-shell dimensional regularization (DR) to regulate both the ultraviolet (UV) and the

infrared (IR) divergences. The choice of IR regulator in SCET is a delicate matter [11], and one has to make sure that the IR regulator accurately generates the IR behavior of the full theory. In Ref. [4], quark off-shellness was used as IR regulator, however all the results can also be obtained by using pure on-shell DR. This was checked explicitly. We will also follow the normalization conventions for the DY and DIS cross sections given in [1]. Thus the Born cross sections are $\delta(1-z)$ and $\delta(1-x)$ respectively.

III.   MATCHINGS AT Q^2 AND $Q^2(1-z)^2$ FOR DY: $p\bar{p} \to l^+l^-X$

In this section, we derive the matching conditions for DY at Q^2 and $Q^2(1-z)^2$ at one-loop order. From these, we re-derive the known one-loop coefficient function in SCET. To make the discussion self-contained, we first review the result for DIS process [4].

A.   Review of DIS in SCET

Since the DY and DIS processes are related to each other, let us review, briefly, how the analysis for DIS in SCET was performed. Choosing the Breit frame, let the incoming quark be $\bar{n}$-collinear, (i.e. moving in the $+z$ with large p_1^+ and small p_1^-) and the outgoing quark be n-collinear, and let $x \equiv Q^2/2p_1 \cdot q$. The invariant mass of the final hadronic state for DIS in the limit $x \to 1$ is: $p_X^2 \cong Q^2(1-x)$. Simple kinematic considerations show that for either collinear or soft gluon radiated into the final state: $p_X^2 \cong Q^2\lambda^2 + O(\lambda^4)$ thus we identify: $\lambda^2 \sim 1 - x$.

The Mellin transform of a function f is defined as: $M_N[f(x)] = \int_0^1 \mathrm{d}x \; x^{N-1} f(x)$. M_N is also known as "N^{th}-moment" of f. The limit $x \to 1$ corresponds to $N \to \infty$, thus one naturally, identifies, $1 - x \sim 1/N$.

At scale Q^2, the final hadronic states may be effectively considered as a massless jet moving in the n direction. One matches the full QCD current (i.e. only incoming and outgoing quarks) onto SCET$_\text{I}$ current given in Eq.(3). The matrix element of the full QCD current in on-shell DR is given by

$$\langle p_2|j^\mu|p_1\rangle = \gamma^\mu \left\{ 1 + \frac{\alpha_s}{4\pi}C_F \left[-\frac{2}{\epsilon_{\text{IR}}^2} - \frac{1}{\epsilon_{\text{IR}}}\left(2\ln\frac{\mu^2}{Q^2}+3\right) - \ln^2\frac{\mu^2}{Q^2} - 3\ln\frac{\mu^2}{Q^2} - 8 + \frac{\pi^2}{6}\right]\right\} \tag{6}$$

The $O(\alpha_s)$ virtual corrections to SCET current are scaleless and vanish in pure DR, thus the matching condition at Q^2 equals the finite part of Eq. (6)

$$C_{\text{DIS}}(\mu) = 1 + \frac{\alpha_s}{4\pi}C_F \left[-\ln^2\frac{\mu^2}{Q^2} - 3\ln\frac{\mu^2}{Q^2} - 8 + \frac{\pi^2}{6}\right] . \tag{7}$$

The anomalous dimension of SCET$_\text{I}$ current satisfies

$$\mu\frac{\mathrm{d}C_{\text{DIS}}(\mu)}{\mathrm{d}\mu} = \gamma_1(\mu) \; C_{\text{DIS}}(\mu) , \tag{8}$$

which gives

$$\gamma_1(\mu) = -\frac{\alpha_s}{4\pi}C_F \left[4\ln\frac{\mu^2}{Q^2} + 6\right] . \tag{9}$$

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As the intermediate scale $Q^2(1-x)$, the final-state invariant mass p_X^2 must be considered as "large", and the jet-like final-state n-collinear hadronic modes can be integrated out. This is done by matching a product of SCET_I currents onto an effective theory (SCET_II) where the n-collinear quark is "replaced" by a soft Wilson line, thus one is led to the matrix element of SCET_II quark operator given in Eq. (5) between quark states. This is very much like the standard operator product expansion. If one computes the matching condition at $Q^2(1-x)$ by using pure on-shell DR, then one has to consider only the contribution of Feynman diagrams where a real gluon is emitted into the final state, because the one-loop virtual diagrams are scaleless and thus vanish.

If one computes the real-gluon emission diagrams in SCET_I, one finds the $O(\alpha_s)$ matching condition (coefficient function) in [4]

$$\mathcal{M}_\text{DIS}(x) = \frac{\alpha_s}{2\pi} C_F \; \theta(0 \leq x \leq 1) \left\{ 2 \left[\frac{\ln(1-x)}{(1-x)} \right]_+ + \left[2\ln\frac{Q^2}{\mu^2} - \frac{3}{2} \right] \frac{1}{(1-x)_+} \right.$$
$$\left. + \left[\ln^2\frac{Q^2}{\mu^2} - \frac{3}{2}\ln\frac{Q^2}{\mu^2} + \frac{7}{2} - \frac{\pi^2}{2} \right] \delta(1-x) \right\} \; . \tag{10}$$

The above result equals exactly to the finite part of the full QCD calculation of the real gluon emission diagrams [1], up to terms that vanish in the large moment limit. Taking the product, $C_\text{DIS}^2(Q)[\delta(1-x) + \mathcal{M}_\text{DIS}(x)]$ yields the full QCD coefficient function. Therefore, the usual one-step matching in DIS is divided into two steps, virtual and real corrections, in SCET. This is also true in Drell-Yan process to be discussed below.

B. Drell-Yan Process

In the Drell-Yan process, the invariant mass of the final hadronic state is

$$p_X^2 = Q^2 \left(1 + \frac{1}{z} - \frac{1}{x_1} - \frac{1}{x_2} \right) \cong Q^2 \left[(1-x_1)(1-x_2) \right] \cong Q^2 \lambda^4 \; , \tag{11}$$

where

$$z = \frac{Q^2}{s}, \quad x_1 = \frac{Q^2}{2p_1 \cdot q}, \quad x_2 = \frac{Q^2}{2p_2 \cdot q} \; . \tag{12}$$

Since $1 - z \sim \lambda^2$, $p_X^2 \sim Q^2(1-z)^2$. To integrate out the final hadronic states, we match at the intermediate scale $Q^2(1-z)^2$.

At the scale Q^2, the final-hadron states effectively have $p_X^2 = 0$. We match the full QCD annihilation current: $q\bar{q} \rightarrow \gamma^*$ onto the SCET_I current. The matching condition can be obtained simply by analytically continuing the full-QCD matrix element in Eq. (6) from space-like $q^2 = -Q^2$ to time-like $q^2 = Q^2$,

$$\langle 0 | j^\mu | p_1 \bar{p}_2 \rangle = \gamma^\mu \left\{ 1 + \frac{\alpha_s}{4\pi} C_F \left[-\frac{2}{\epsilon_\text{IR}^2} - \frac{1}{\epsilon_\text{IR}} \left(2\ln\frac{\mu^2}{Q^2} + 3 \right) \right. \right.$$
$$\left. \left. - \ln^2\frac{\mu^2}{Q^2} - 3\ln\frac{\mu^2}{Q^2} - 8 + \frac{7\pi^2}{6} \right] \right\} \; . \tag{13}$$

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The matching condition at Q^2 is thus

$$C_{\text{DY}}(\mu) \;=\; 1 + \frac{\alpha_s}{4\pi} C_F \left[-\ln^2 \frac{\mu^2}{Q^2} - 3\ln\frac{\mu^2}{Q^2} - 8 + \frac{7\pi^2}{6} \right] . \tag{14}$$

The anomalous dimension is the same as Eq. (9). It should be noted that ϵ_{IR}-dependent terms in Eq. (13) equal the negative of the ϵ_{UV} terms in the effective theory. This is so since, as mentioned earlier, the SCET virtual contributions vanish in pure on-shell DR due to cancellation of the form $\frac{1}{\epsilon_{\text{UV}}} - \frac{1}{\epsilon_{\text{IR}}}$, and because the IR behavior of the effective theory must match that of the full QCD. Thus the UV counter-term for the current in the effective theory equals

$$\text{c.t.} \;=\; \frac{\alpha_s}{2\pi} C_F \left[-\frac{1}{\epsilon^2} - \frac{3}{2\epsilon} - \frac{1}{\epsilon}\ln\frac{\mu^2}{Q^2} \right] . \tag{15}$$

Below Q^2, the DY process is described by SCET$_{\text{I}}$. Because the virtual diagrams vanish in on-shell DR, we need only consider real emission diagrams given in Figs. 1 and 2, where the cut lines go through the real gluon in the final state. Let us take, in the c.o.m. frame, the momentum k^μ of the emitted gluon as:

$$k^\mu = (|k|, ... |k|\cos\theta) , \tag{16}$$

and introduce the Mandelstam variables (besides s)

$$t \;=\; -\frac{Q^2}{z}(1-z)(1-y), \quad u = -\frac{Q^2}{z}(1-z)y , \tag{17}$$

where: $y = \frac{1}{2}(1+\cos\theta)$. The calculation of the contributions from the above diagrams is performed by taking the square of the amplitude for emitting a real gluon and then integrating over the phase space for the production of a massive photon [1]

$$\text{PS} \;=\; \frac{1}{8\pi}\left(\frac{4\pi}{Q^2}\right)^\epsilon \frac{1}{\Gamma(1-\epsilon)} z^\epsilon (1-z)^{1-2\epsilon} \int_0^1 \mathrm{d}y\, (y(1-y))^{-\epsilon} . \tag{18}$$

Diagrams (c) in Fig.1 and (a)-(b) in Fig. 2 are zero due to $n^2 = \bar{n}^2 = 0$. Diagrams (a) and (b) have the same contribution as their full QCD counterparts. The sum of the three remaining contributions completes the total result to that of full QCD which can be found in, e.g, [1]. We perform the phase space integration for the sake of completeness, keeping only singular terms in the limit $z \to 1$,

$$\begin{aligned}
\mathcal{I}_{\text{DY}}(z) &= \frac{\alpha_s}{2\pi} C_F\, \theta(0 \le z \le 1) \left(\frac{\mu^2 e^\gamma}{Q^2}\right)^\epsilon \frac{z^\epsilon (1-z)^{1-2\epsilon}}{\Gamma(1-\epsilon)} \int_0^1 \mathrm{d}y\, [y(1-y)]^{-\epsilon} \times \frac{2}{y(1-y)}\frac{1}{(1-z)^2} \\
&= \frac{\alpha_s}{2\pi} C_F\, \theta(0 \le z \le 1) \times 2\left[\frac{1}{\epsilon^2}\delta(1-z) - \frac{2}{\epsilon}\left(\frac{1}{1-z}\right)_+ + 4\left(\frac{\ln(1-z)}{1-z}\right)_+ \right] \\
&\quad \times \left[1 + \epsilon\ln\frac{\mu^2}{Q^2} + \epsilon^2\left(\frac{1}{2}\ln^2\frac{\mu^2}{Q^2} - \frac{\pi^2}{4}\right) \right] \\
&= \frac{\alpha_s}{2\pi} C_F\, \theta(0 \le z \le 1)\left[\frac{2}{\epsilon^2}\delta(1-z) + \frac{2}{\epsilon}\ln\frac{\mu^2}{Q^2}\delta(1-z) - \frac{4}{\epsilon}\left(\frac{1}{1-z}\right)_+ \right. \\
&\quad \left. + 8\left(\frac{\ln(1-z)}{1-z}\right)_+ - 4\ln\frac{\mu^2}{Q^2}\left(\frac{1}{1-z}\right)_+ + \delta(1-z)\left(\ln^2\frac{\mu^2}{Q^2} - \frac{\pi^2}{2}\right) \right] .
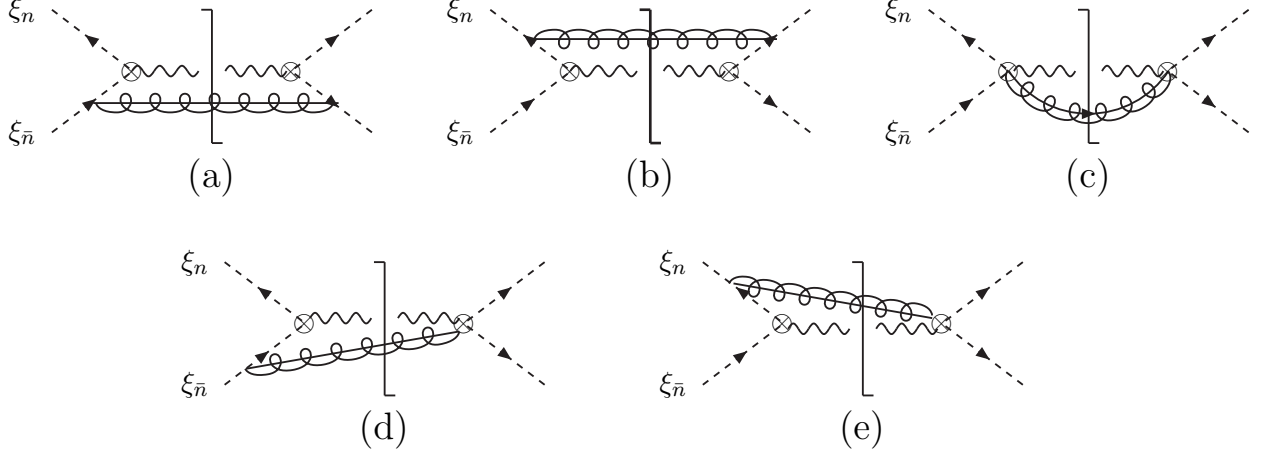\end{aligned} \tag{19}$$

FIG. 1: *Collinear real-gluon contributions to DY crosssection in SCET. Diagram (c) represents the two possibilities of n-collinear and $\bar{n}$-collinear gluons. Diagrams (d) and (e) have mirror image graphs.*
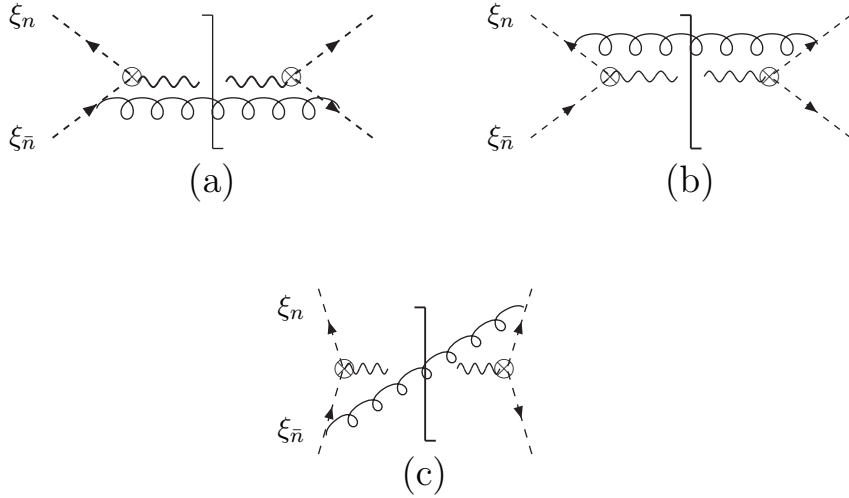

FIG. 2: *Real soft-gluon contributions to DY cross section in SCET. Diagram (c) has mirror image graph.*

In the above calculation we ignored the contribution from $\ln z/(1-z)$ since this function is regular in the limit $z \to 1$ and the corresponding Mellin moments vanish at large N. As expected, the result is identical to the full QCD calculation of the real gluon contributions to the DY differential cross section in the limit $z \to 1$ [1]. The ϵ-terms in Eq. (19) are a combination of UV and IR contributions.

To see how to interpret these terms, we need to consider the PDF given in Eq. (5). In pure on-shell DR, the $O(\alpha_s)$ corrections to the unrenormalized parton distribution vanish due to scaleless integrals. The UV-behavior of the SCET PDF is given by [4]: $\frac{\alpha_s}{2\pi \epsilon_{\mathrm{UV}}} P_{q \leftarrow q}(z)$

where in the limit $z \to 1$

$$P_{q \leftarrow q} = C_F \left[\frac{2}{(1-z)_+} + \frac{3}{2} \delta(1-z) \right] \theta(0 \leq z \leq 1) , \tag{20}$$

which is the well-known Altarelli-Parisi splitting kernel. Thus the renormalized parton distribution is $-\frac{\alpha_s}{2\pi\epsilon_{\mathrm{IR}}} P_{q \leftarrow q}(z)$. Writing the ϵ-terms in Eq. (19) as

$$\frac{\alpha_s}{2\pi} C_F \ \theta(0 \leq z \leq 1) \left[\left(\frac{2}{\epsilon^2} + \frac{3}{\epsilon} + \frac{2}{\epsilon} \ln \frac{\mu^2}{Q^2} \right) \delta(1-z) - \frac{4}{\epsilon} \left(\frac{1}{1-z} \right)_+ - \frac{3}{\epsilon} \delta(1-z) \right] .$$

Taking into account the counter-term for the SCET current, Eq. (15), and matching the DY cross section onto a product of two PDFs, all the ϵ-dependent terms cancel.

The finite part of Eq. (19) yields $O(\alpha_s)$ matching condition (coefficient function)

$$\mathcal{M}_{\mathrm{DY}}(z) = \frac{\alpha_s}{2\pi} C_F \ \theta(0 \leq z \leq 1) \left[8 \left(\frac{\ln(1-z)}{1-z} \right)_+ - 4 \ln \frac{\mu^2}{Q^2} \left(\frac{1}{1-z} \right)_+ \right.$$
$$\left. + \delta(1-z) \left(\ln^2 \frac{\mu^2}{Q^2} - \frac{\pi^2}{2} \right) \right] . \tag{21}$$

The moments of the matching condition at $Q^2(1-z)^2$, Eq. (21), are

$$\mathcal{M}_{\mathrm{DY}}(N) = \frac{\alpha_s}{2\pi} C_F \left[\ln^2 \frac{\overline{N}^2 \mu^2}{Q^2} + \frac{\pi^2}{6} \right] , \tag{22}$$

where $\overline{N} = N \exp(\gamma_E)$ and γ_E is the Euler constant. In order to minimize the large logs we choose $\mu = Q/\overline{N}$. This is different from the DIS case where at the intermediate scale we set $\mu = Q/\sqrt{\overline{N}}$. Thus for DY we identify $1 - z \sim 1/N$. At this stage it is appropriate to clarify the following point: the above calculation demonstrates the all order factorization theorem [9, 12] of the DY process to first order in α_s, i.e, all the IR divergences are contained into a product of two PDFs defined in Eq.(5).

Below the scale $Q^2(1-z)^2$, we have SCET$_{\mathrm{II}}$ in which the all hard scales have been integrated out. The running of the moments of the PDF with μ is governed by the well-known anomalous dimension $-\gamma_{2,N}$ [4, 13] where

$$\gamma_{2,N} = -\frac{\alpha_s}{2\pi} C_F \left[4 \sum_{j=2}^{N} \frac{1}{j} - \frac{2}{N(N+1)} + 1 \right] , \tag{23}$$

In [4] it was shown, to first order in α_s, that the running of the PDF comes from Fenyman diagrams where only collinear quarks and gluons interact and the soft contribution (or usoft-in the terminology of Ref.[4]) vanishes. This is of-course consistent with the factorization theorem of DY which we assume that it holds to all orders in perturbation theory.
In the large N limit we have

$$\gamma_{2,N} \cong -\frac{\alpha_s}{2\pi} C_F \left[4 \ln \overline{N} - 3 \right] . \tag{24}$$

At the lowest scale μ_0, one identifies the PDF moments as the matrix elements of local twist operators $A_N(\mu_0)$ [14].

IV. RESUMMATION AND EXPONENTIATION

By now, we have obtained all the ingredients we need to write down an expression for the moments of the DY differential cross section ($\frac{d\sigma_{DY}}{dQ^2}$). By taking the square of the current matching coefficient at $\mu^2 = Q^2$, running down the scale with anomalous dimension $2\gamma_1$ to $\mu^2 = Q^2/\overline{N}^2$, we then multiply with the moments of matching condition into the product of the matrix element of PDF. Then we run down the scale to $\mu \simeq \Lambda_{\rm QCD}$ with $2\gamma_2$ and match onto the moments of local twist-two operators. In summary, one gets

$$\left(\frac{d\sigma_{\rm DY}}{dQ^2}\right)_N = C_{\rm DY}^2(Q)e^{-I_{\rm DY1}}\left(1 + \mathcal{M}_{\rm DY}(N, \mu = Q/\overline{N})\right)e^{-I_{\rm DY2}}A_N^2(\mu_0) , \tag{25}$$

where

$$I_{\rm DY1} = \int_{Q/\overline{N}}^{Q}\frac{d\mu}{\mu}2\gamma_1(\mu); \qquad I_{\rm DY2} = \int_{\mu_0}^{Q/\overline{N}}\frac{d\mu}{\mu}2\gamma_2(\mu) , \tag{26}$$

where $\gamma_1(\alpha_s, Q^2/\mu^2)$ is the anomalous dimension of the SCET current and $\gamma_2(\alpha_s, N)$ is the anomalous dimension of the twist-two operator.

The above can be compared with the DIS result,

$$F_{2N}(Q^2) = C_{\rm DIS}^2(Q)e^{-I_{\rm DIS1}}\left(1 + \mathcal{M}_{\rm DIS}(N, \mu = Q/\sqrt{\overline{N}})\right)e^{-I_{\rm DIS2}}A_N(\mu_0) \tag{27}$$

where

$$I_{\rm DIS1} = \int_{Q/\sqrt{\overline{N}}}^{Q}\frac{d\mu}{\mu}2\gamma_1(\mu); \qquad I_{\rm DIS2} = \int_{\mu_0}^{Q/\sqrt{\overline{N}}}\frac{d\mu}{\mu}\gamma_2(\mu) . \tag{28}$$

Let us define a physical observable which is a ratio of the moments between DY and DIS squared,

$$\Delta_N = \frac{(d\sigma_{DY}/dQ^2)_N}{F_{2N}^2} . \tag{29}$$

At one-loop order, one has

$$\Delta_N = 1 + \frac{\alpha_s}{2\pi}C_F\left(2\ln^2\overline{N} - 3\ln\overline{N} + 1 + \frac{5}{3}\pi^2\right) , \tag{30}$$

which is consistent with the known result. When summing over higher-order corrections of form $\alpha_s^n\ln^m\overline{N}$ with $m \leq 2n$, Δ_N can be expressed in terms of an exponential form, $\exp(f)$, with $f = \ln\overline{N}f_{-1}(\alpha_s\ln\overline{N}) + f_0(\alpha_s\ln\overline{N}) + \alpha_sf_1(\alpha_s\ln\overline{N}) + \dots$. The first term yields the leading double-logarithm resummation, and the second gives the next-to-leading logarithm resummation (NLL), etc. In the present calculation, we can have an accuracy up to NLL.

To NLL resummation, Δ_N can be expressed as

$$\ln\Delta_N = \int_{Q/\sqrt{\overline{N}}}^{Q}2\gamma_1(\alpha_s(\mu))\frac{d\mu}{\mu} + \int_{Q/\overline{N}}^{Q/\sqrt{\overline{N}}}2\left[\gamma_1(\alpha_s(\mu)) - \gamma_2(\alpha_s(\mu))\right]\frac{d\mu}{\mu} , \tag{31}$$

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where the anomalous dimensions have the form

$$
\begin{aligned}
\gamma_1 &= \Gamma_1 \ln Q^2/\mu^2 + \Gamma_2 + \Gamma_3/2 \ , \\
\gamma_2 &= 2\Gamma_1 \ln \overline{N} + \Gamma_3 \ ,
\end{aligned}
\tag{32}
$$

to all orders in perturbation theory. To NLO, one has $\Gamma_3 = 2\Gamma_2$, and the above expression can be expressed as

$$
\ln \Delta_N = 2 \int_{\overline{N}^{-1}}^{1} \frac{dy}{y} \left[\int_{yQ}^{\sqrt{\overline{y}}Q} 2\Gamma_1(\alpha_s(\mu)) \frac{d\mu}{\mu} + \Gamma_2(\alpha_s(\sqrt{\overline{y}}Q)) \right] \ .
\tag{33}
$$

This result does not have an infrared renormalon or Landau singularity because the expression is manifestly valid only when the scale $Q/\overline{N}$ is much larger than Λ_{QCD}. If one takes the limit $N \to \infty$ with a fixed Q, the above result can also be expressed as

$$
\Delta_N = \exp \left[-2 \int_0^1 dz \frac{z^{N-1} - 1}{1 - z} \left(\int_{Q^2(1-z)}^{Q^2} \Gamma_1(\alpha_s(k^2)) \frac{dk^2}{k^2} + \Gamma_2 \left(\alpha_s((1-z)Q^2) \right) \right) \right]
\tag{34}
$$

which is the same as what has been obtained by Catani and Trentadue [3], and Sterman [2]. This expression has the infrared renormalon problem because the integral covers the soft momentum region where the strong coupling constant is ill-defined.

It is instructive to recall how the resummation in the large-x limit is achieved in Sterman's approach [2]. The DY cross section and DIS structure function can be factorized in $x \to 1$ limit into a product of *hard parts*, a *soft part*, and *collinear parts*. Different parts may be gauge-dependent, but the product is not. The renormalization group equations for individual parts can be derived straightforwardly.

The hard part contains lines with momentum of order Q and is included in the first stage matching in SCET approach. The soft part is defined as the matrix element of Wilson lines and is infared finite (for a definition in SCET, see [15]). This soft contribution comes in fact from gluon lines of virtuality of order $Q^2(1-x)$, which can be calculated in perturbation theory when $Q^2(1-x) \gg \Lambda_{\mathrm{QCD}}^2$ [2]. In SCET approach, this contribution is taken into account by the second stage matching. The collinear parts are either special parton distributions or jet functions or both. They depend on a rapidity cut-off and obey an evolution equation in rapidity. This equation is quite general, and when solved, contains the resummed double logarithms. When $Q^2(1-x) \gg \Lambda_{\mathrm{QCD}}^2$, the special parton distriubtions can be factorized in terms of ordinary parton distributions, whereas the jet functions can be calculated entirely in pQCD.

V.   CONCLUSION

Following a previous work on summing large logarithms as $x \to 1$ in DIS, we use the same SCET formalism to study the resummation in DY. The steps are quite similar and the result is shown in Eq. (25). The result is consistent with the known result, but is free of the Landau singularity.

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